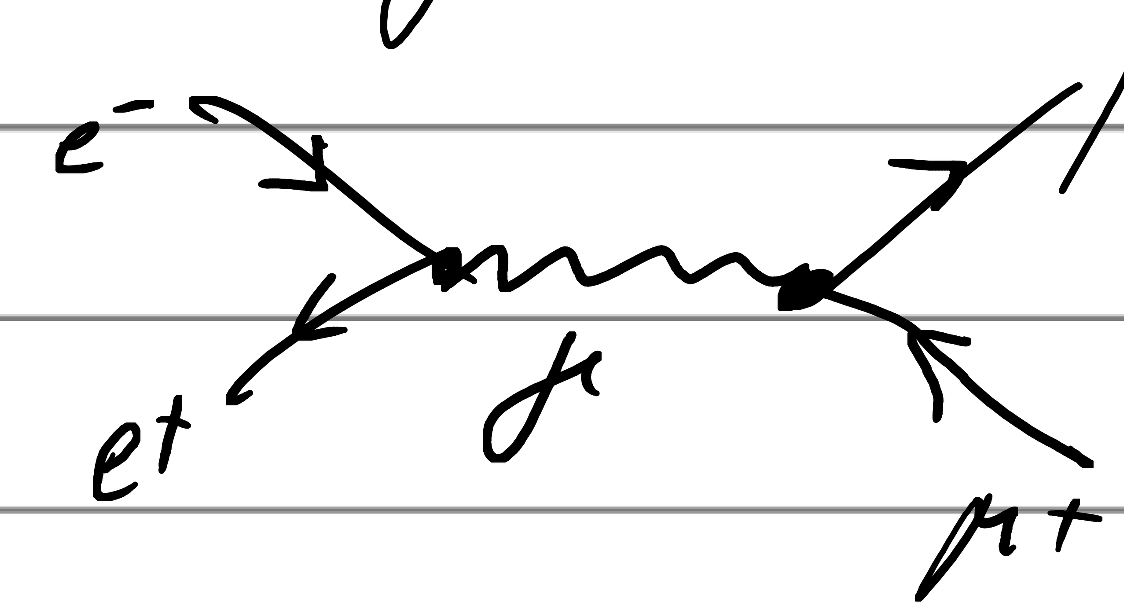
Lecture 8
Electron-positron annihilation

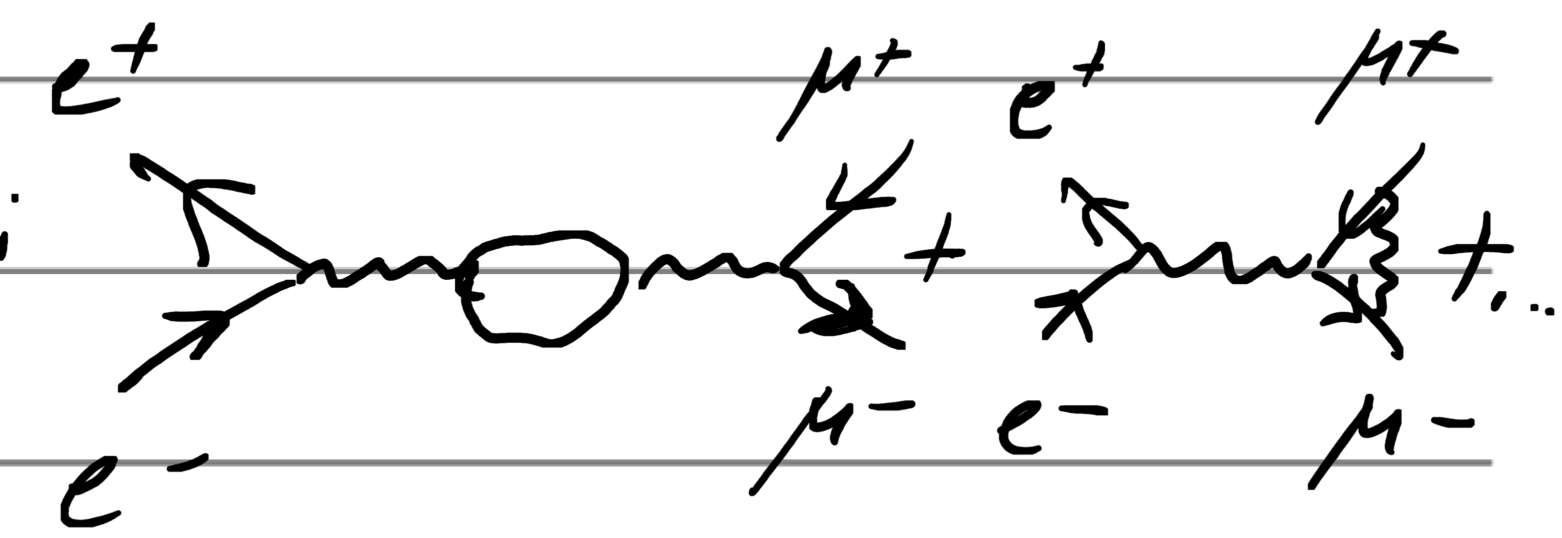
How to calculate QED cross section

1. Draw all possible Feynman diagrams (for $e^+e^- \to \mu^+\mu^-$)

   - lowest order: 1 diagram
     $M \propto e^2 \propto \alpha_{em}$



   - many second order diagrams:
     $M \propto e^4 \propto \alpha_{em}^2$



2. For each diagram calculate the matrix element using Feynman rules (draw the rules on the board)

3. Sum the individual Matrix elements: $M_{fi} = M_1 + M_2 + \dots$
   (interferences both + and − possible)

4. Square the matrix element: $|M_{fi}|^2 = (M_1^* + M_2^* + \dots)(M_1 + M_2 + \dots)$
   giving the full perturbative expansion in α_{em}.
   For $e^+e^- \to \mu^+\mu^-$ Leading Order QED computation accurate to 1%!

5. Calculate the decay rate/cross section using

- decay width (rate): $\Gamma = \frac{|\vec{p}^*|}{32\pi^2 m_i^2} \int |M_{fi}|^2 d\Omega$

- scattering in CoM frame: $\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f^*|}{|\vec{p}_i^*|} \cdot |M_{fi}|^2$

- scattering lab. frame: $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{M \cdot E_1}\right)^2 |M_{fi}|^2$

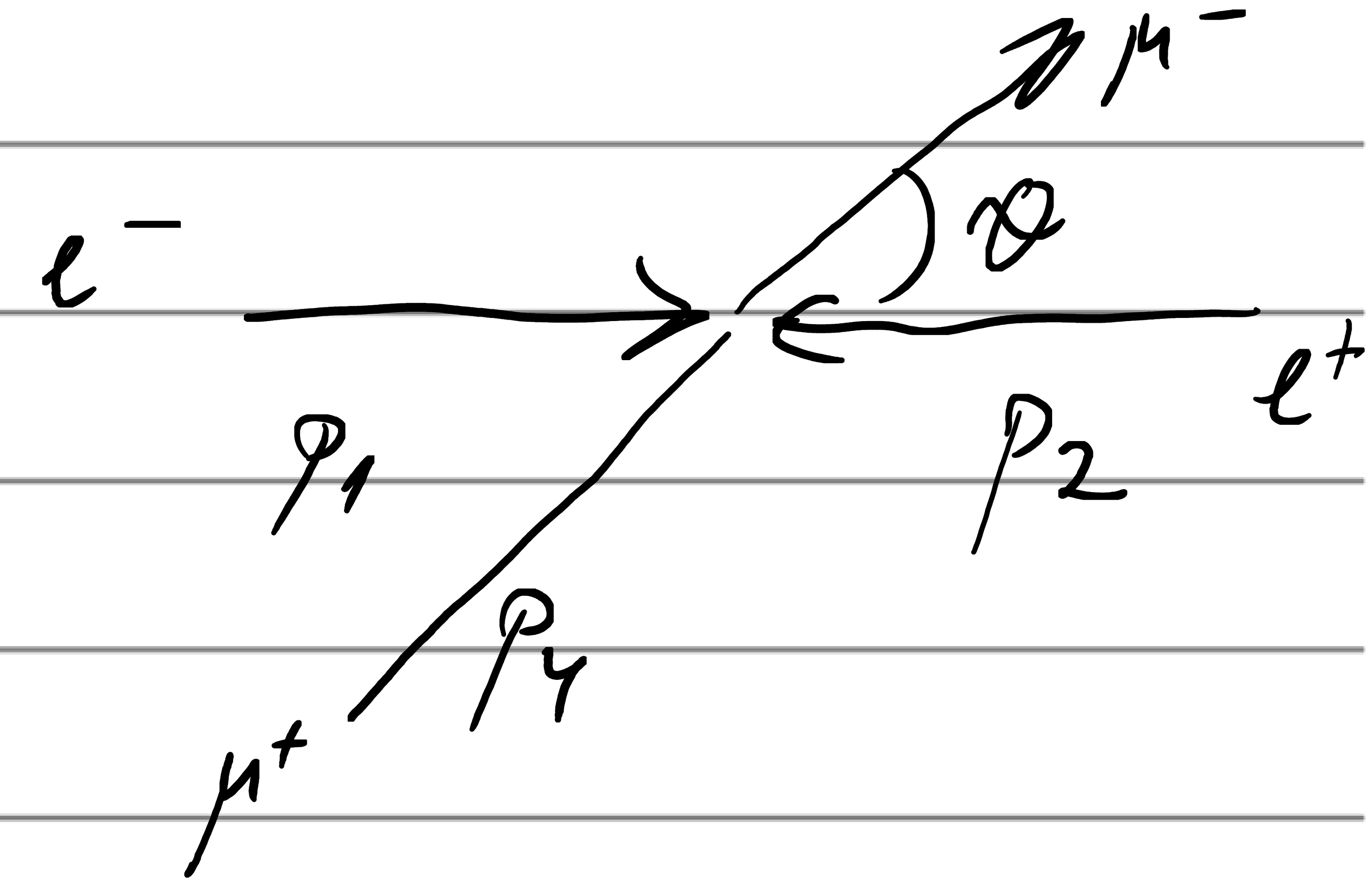
If we consider only LO and Next-to-leading order (NLO) we get:

$$|M_{fi}|^2 = \left(\alpha M_{LO} + \alpha^2 \sum_j M_{NLO,j}\right)\left(\alpha M_{LO}^* + \alpha^2 \sum_k M_{NLO,k}^*\right) =$$

$$= \alpha^2 |M_{LO}|^2 + \alpha^3 \sum_j \left(M_{LO} \cdot M_{NLO,j}^* + M_{LO}^* M_{NLO,j}\right) + \alpha^4 \sum_{j,k} M_{NLO,j} \cdot M_{NLO,k}^*$$

$\Rightarrow$ Any potential higher than LO term is suppressed by $\alpha = 1/137 \approx 0{,}7\%$ $\Rightarrow$ using only LO diagram we can expect QED calculations to be accurate to $O(1\%)$

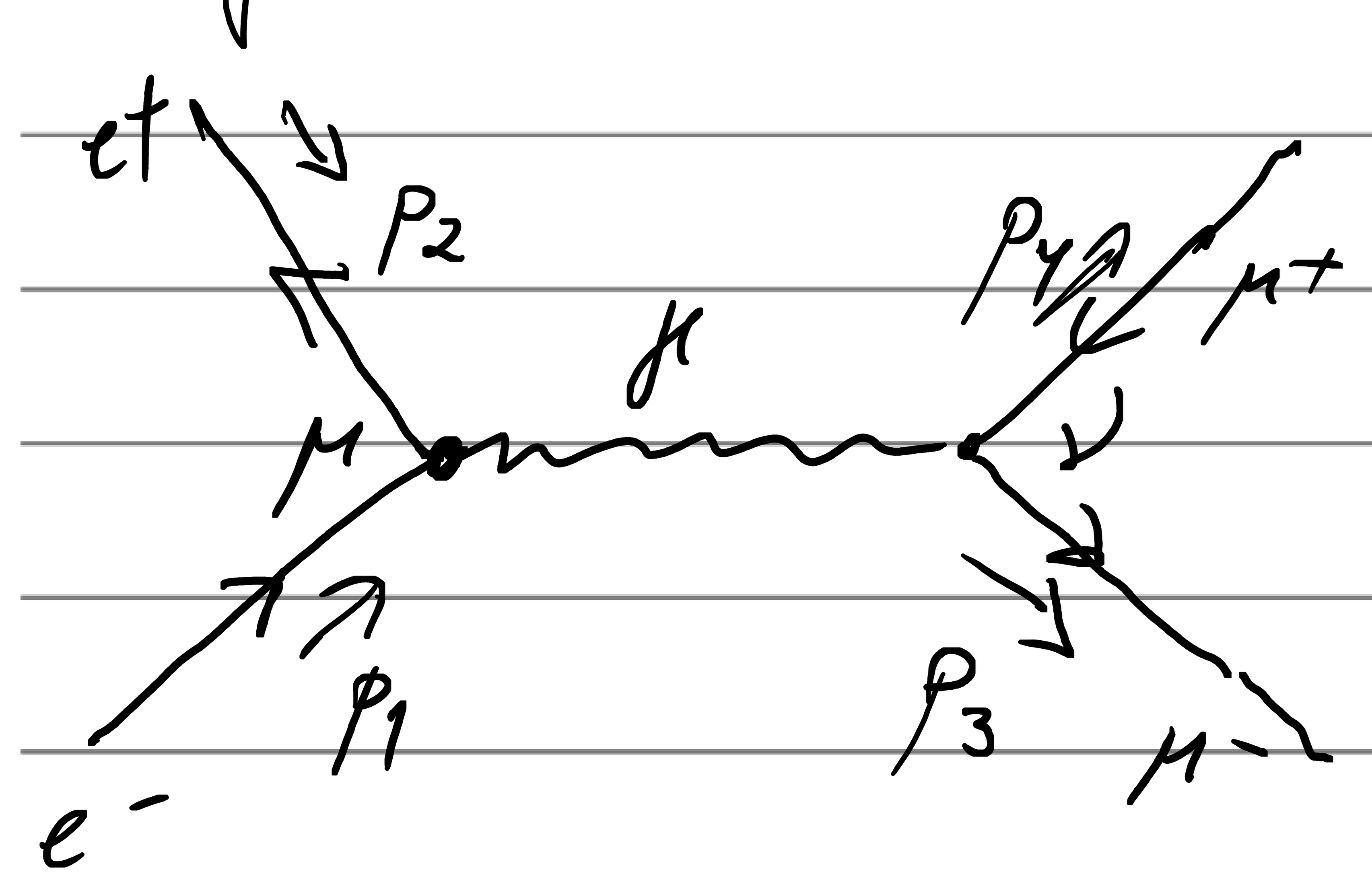
$e^+e^- \to \mu^+\mu^-$



$p_1 = (E, 0, 0, p)$, $p_2 = (E, 0, 0, -p)$

$p_3 = (E, \vec{p}_f)$, $p_4 = (E, -\vec{p}_f)$

Diagram (LO):



$$iM = \underbrace{\bar{v}(p_2)\, ie\gamma^\mu u(p_1)}_{j^\mu_{(e)}} \cdot \left(\frac{-i g_{\mu\nu}}{q^2}\right) \cdot \underbrace{\bar{u}(p_3)\, ie\gamma^\nu v(p_4)}_{j^\nu_{(\mu)}}$$

$$q^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2 = 4E^2 = s$$

$$iM = -\frac{e^2}{q^2} \cdot i g_{\mu\nu} j^\mu_{(e)} j^\nu_{(\mu)} = -\frac{e^2}{s} \cdot i g_{\mu\nu} j^\mu_{(e)} j^\nu_{(\mu)} = -\frac{ie^2}{2\sqrt{E}} g_{\mu\nu} j^\mu_{(e)} j^\nu_{(\mu)}$$

Lorentz-invariant quantity

We need to consider the spin of e^+e^-, $\mu^+\mu^-$

- in total 16 combinations (2^4)
- each combination represents a physical process
- we must sum over all 16 possible combinations + average over the initial helicity states

The helicity states are orthogonal $\Rightarrow$ no interference $\Rightarrow$ $\Rightarrow$ we can consider each combination separately

For each initial state helicity configuration the cross section is just the sum over all final states

e.g. $\sum |M_{RR}|^2 = |M_{RR\to RR}|^2 + |M_{RR\to RL}|^2 + |M_{RR\to LR}|^2 + |M_{RR\to LL}|^2$

Typically in e^+e^- colliders e^+e^- are unpolarised $\Rightarrow$ equal number of $h=+1$ and $h=-1$ configurations in the initial state

$\Rightarrow$ spin-averaged summed matrix element:

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |M|^2$$

Helicity eigenstates: $N = \sqrt{E+m}$, $c = \cos\theta/2$ $s = \sin\theta/2$

$$u_\uparrow = N \begin{pmatrix} c \\ s e^{i\varphi} \\ \frac{p}{E+m} \cdot c \\ \frac{p}{E+m} \cdot s e^{i\varphi} \end{pmatrix}, \quad u_\downarrow = N \begin{pmatrix} -s \\ c e^{i\varphi} \\ \frac{p}{E+m} s \\ \frac{-p}{E+m} \cdot c e^{i\varphi} \end{pmatrix}, \quad v_\uparrow = N \begin{pmatrix} \frac{p}{E+m} s \\ \frac{-p}{E+m} \cdot c e^{i\varphi} \\ -s \\ c e^{i\varphi} \end{pmatrix}, \quad v_\downarrow = N \begin{pmatrix} \frac{p}{E+m} \cdot c \\ \frac{p}{E+m} \cdot s e^{i\varphi} \\ c \\ s e^{i\varphi} \end{pmatrix}$$

$E \gg m \downarrow$ $\quad E \gg m \downarrow$ $\quad E \gg m \downarrow$ $\quad E \gg m \downarrow$

$$u_\uparrow = \sqrt{E} \begin{pmatrix} c \\ s e^{i\varphi} \\ c \\ s e^{i\varphi} \end{pmatrix}, \quad u_\downarrow = \sqrt{E} \begin{pmatrix} -s \\ c e^{i\varphi} \\ s \\ -c e^{i\varphi} \end{pmatrix}, \quad v_\uparrow = \sqrt{E} \begin{pmatrix} s \\ -c e^{i\varphi} \\ -s \\ c e^{i\varphi} \end{pmatrix}, \quad v_\downarrow = \sqrt{E} \begin{pmatrix} c \\ s e^{i\varphi} \\ c \\ s e^{i\varphi} \end{pmatrix}$$

In the limit $\sqrt{s} \gg m_\mu$ the four-momenta are:

$$p_1 = (E, 0, 0, E), \quad p_2 = (E, 0, 0, -E)$$

$$p_3 = (E, E\sin\theta, 0, E\cos\theta), \quad p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

We can take that μ^-, μ^+ are produced with $\varphi = 0$ and $\varphi = \pi$ respectively

For the initial state electron:
- two possible spinors for $(\theta = 0, \varphi = 0)$ electron
- two possible spinors for $(\theta = \pi, \varphi = \pi)$ positron

$$\underbrace{u_\uparrow(p_1) = \sqrt{E}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \quad u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}}_{e^-}, \quad v_\uparrow(p_2) = \sqrt{E}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}, \quad v_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix}$$

Similarly for $\mu^+\mu^-$: $(\theta, 0)$ for μ^- and $(\pi - \theta, \pi)$ for μ^+

using $\sin\left(\frac{\pi - \theta}{2}\right) = \cos\theta/2$ $\cos\left(\frac{\pi - \theta}{2}\right) = \sin\theta/2$ and $e^{i\pi} = -1$

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix}c\\s\\c\\s\end{pmatrix}, \quad u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix}, \quad v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix}c\\s\\-c\\-s\end{pmatrix}, \quad v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}s\\-c\\s\\-c\end{pmatrix}$$

Muon current: $j_{(\mu)}^{\nu} = \bar{u}(p_3)\gamma^\nu v(p_4)$ for all helicity combinations

$$\bar{\psi}\gamma^0\phi = \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4$$

$$\bar{\psi}\gamma^1\gamma^0\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1$$

$$\bar{\psi}\gamma^2\gamma^0\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1)$$

$$\bar{\psi}\gamma^3\gamma^0\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2$$

$$j_\mu^0 = \bar{u}_\uparrow(p_3)\gamma^0 v_\downarrow(p_4) = E(cs - sc + cs - sc) = 0$$

$\cos\theta = \sin^2\theta/2 - \cos^2\theta/2$

$$j_\mu^1 = \bar{u}_\uparrow(p_3)\gamma^1 v_\downarrow(p_4) = E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) = -2E\cos\theta$$

$$j_\mu^2 = \bar{u}_\uparrow(p_3)\gamma^2 v_\downarrow(p_4) = -iE(-c^2 - s^2 - c^2 - s^2) = 2iE$$

$$j_\mu^3 = \bar{u}_\uparrow(p_3)\gamma^3 v_\downarrow(p_4) = E(cs + cs + cs + sc) = 2E\sin\theta$$

$$\Rightarrow j_{(\mu)RL}^{\nu} = \bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4) = 2E(0, -\cos\theta, i, \sin\theta) \rightarrow \mu_R^- \mu_L^+$$

$$j_{(\mu)RR}^{\nu} = \bar{u}_\uparrow(p_3)\gamma^\nu v_\uparrow(p_4) = (0,0,0,0)$$

due to $E \gg m_\mu$ limit

$$j_{(\mu)LL}^{\nu} = \bar{u}_\downarrow(p_3)\gamma^\nu v_\downarrow(p_4) = (0,0,0,0)$$

$$j_{(\mu)LR}^{\nu} = \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) = 2E(0, -\cos\theta, -i, +\sin\theta) \rightarrow \mu_L^- \mu_R^+$$

Electron current:

$(AB)^\dagger = B^\dagger A^\dagger$

$\gamma^{0\dagger} = \gamma^0$

$\gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu$

$j^\mu_{(e)} = \bar{v}(p_2)\gamma^\mu u(p_1) \qquad j^\nu_{(\mu)} = \bar{u}(p_3)\gamma^\mu v(p_4)$

Relating $j^\nu_{(\mu)}$ to $j^\mu_{(e)}$:

$$[\bar{u}(p_3)\gamma^\nu v(p_4)]^\dagger = v^\dagger(p_4)\gamma^{\nu\dagger}\gamma^{0\dagger}u(p_3) = \bar{v}(p_4)\gamma^\nu u(p_3)$$

$$\Rightarrow \bar{v}_\downarrow(p_4)\gamma^\nu u_\uparrow(p_3) = [\bar{u}_\uparrow(p_3)\gamma^\nu v_\downarrow(p_4)]^* = 2E(0, -\cos\theta, -i, \sin\theta)$$

$$\bar{v}_\uparrow(p_4)\gamma^\nu u_\downarrow(p_3) = [\bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4)]^* = 2E(0, -\cos\theta, +i, \sin\theta)$$

For electrons we simply set $\theta = 0$

$$e^-_R e^+_L = \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_3) = 2E(0, -1, -i, 0)$$

$$e^-_L e^+_R = \bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_3) = 2E(0, -1, +i, 0)$$

We're left with 4 non-vanishing combinations:

$$e^-_R e^+_L : 2E(0,-1,-i,0) \to \mu^-_L \mu^+_R = 2E(0,-\cos\theta,-i,\sin\theta)$$

We call it $\underline{M_{RL}}$ $\Rightarrow |M_{RL}|^2 = \left(+\frac{e^2}{\cancel{4E^2}} \cdot \cancel{4E^2}(1-\cos\theta)\right)^2 = (4\pi\alpha)^2(1-\cos\theta)^2$

using $\frac{e^2}{4\pi} = \alpha \simeq 1/137$

$$|M_{RL}|^2 = |M_{LR}|^2$$

$$e^-_L \mu^+_R \to e^-_R \mu^+_L$$

Let's take M_{RR}:

$$e^-_R e^+_L : 2E(0,-1,-i,0) \to \mu^-_R \mu^+_L : 2E(0,-\cos\theta, i, \sin\theta)$$

$$|M_{RR}|^2 = (-e^2(1+\cos\theta))^2 = (4\pi\alpha)^2(1+\cos\theta)^2 = |M_{LL}|^2$$

$$e^-_L e^+_R \to \mu^-_L \mu^+_R$$

Summing over final states and averaging over initial states

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \cdot \frac{1}{64\pi^2 s}\left(|M_{RR}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{LL}|^2\right) =$$

$$= \frac{(4\pi\alpha)^2}{256\pi^2 s}\left(2(1+\cos\theta)^2 + 2(1-\cos\theta)^2\right) = \boxed{\frac{\alpha^2}{4s}(1+\cos\theta)}$$

Total cross section

$$\sigma = \frac{\alpha^2}{4s} \int (1+\cos^2\theta)\, d\Omega = \frac{\alpha^2}{2\cancel{4}s} \cdot 2\pi \int_{-1}^{+1} (1+\cos^2\theta)\, d(\cos\theta) = \frac{4\pi\alpha^2}{3s}$$

Lorentz-invariant form of the matrix element:

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4} \times \left(|M_{RR}|^2 + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}|^2 \right) = \frac{1}{4} e^4 \left(2(1+\cos\theta)^2 + \right.$$

$$\left. + 2(1-\cos\theta)^2 \right) = e^4 (1+\cos^2\theta)$$

In CoM: $p_1 = (E, 0, 0, E)$, $p_2 = (E, 0, 0, -E)$

$p_3 = (E, E\sin\theta, 0, E\cos\theta)$, $p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$

$p_1 \cdot p_2 = 2E^2$, $p_1 \cdot p_3 = E^2(1-\cos\theta)$, $p_1 \cdot p_4 = E^2(1+\cos\theta)$

$$\Rightarrow \boxed{\langle |M_{fi}|^2 \rangle = 2e^4 \frac{(p_1 p_3)^2 + (p_1 \cdot p_4)^2}{(p_1 \cdot p_2)^2} = 2e^4 \left(\frac{t^2+u^2}{s^2} \right)}$$

$$s = (p_1+p_2)^2 \approx 2p_1 \cdot p_2 \quad (m_1^2 \approx m_2^2 \approx 0)$$
$$t = (p_1-p_3)^2 \approx -2p_1 \cdot p_3 \quad (m_1^2 \approx m_3^2 \approx 0)$$
$$u = (p_1-p_4)^2 \approx -2p_1 \cdot p_4 \quad (m_1^2 \approx m_4^2 \approx 0)$$

Spin in e^+e^- annihilation

Define the z-axis to be along the direction of the incoming e^- beam

- the combined spin of the e^+e^- system is +1 or −1

$\Rightarrow$ non-0 M_{fi} corresponds to e^+e^- colliding in a spin-1 state

RL helicity combination $\rightarrow |S, S_z\rangle = |1, +1\rangle$
LR helicity combination $\rightarrow |S, S_z\rangle = |1, -1\rangle$ | for the e^+e^-

For $\mu^+\mu^-$ is similar: $|1, \pm 1\rangle_\theta$ (measured wrt the axis in the direction of μ^-)

The operator corresponding to the component of spin along an axis defined by a unit vector $\vec{n}$ is

$$\hat{S}_n = \frac{1}{2}\vec{n}.\vec{\sigma}$$

We can use $\hat{S}_n$ to express the $\mu^+\mu^-$ states in terms of the eigenstates of z: e.g. $\mu^-_R\mu^+_L$

$$|1, +1\rangle_\theta = \frac{1}{2}(1-\cos\theta)|1, -1\rangle + \frac{1}{\sqrt{2}}\sin\theta|1, 0\rangle + \frac{1}{2}(1+\cos\theta)|1, +1\rangle$$

$|M_{LR}| \propto \langle\psi|1, -1\rangle$ $\qquad$ $|M_{RR}| \propto \langle\psi|1, +1\rangle$

Chirality

In the limit $E \gg m$ only 4 of the 16 helicity combinations are possible

We can introduce chirality via the γ^5 matrix

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Properties: $(\gamma^5)^2 = I$, $\gamma^{5\dagger} = \gamma^5$, $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$

When $E \gg m$ helicity eigenstates are also eigenstates of γ^5:

$$\gamma^5 u_\uparrow = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \cdot \sqrt{E} = \sqrt{E} \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = + u_\uparrow$$

$$\gamma^5 u_\downarrow = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \sqrt{E} \begin{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ -1 \end{pmatrix} \end{pmatrix} = \sqrt{E} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} = - u_\downarrow$$

$$\gamma^5 v_\uparrow = - v_\uparrow, \quad \gamma^5 v_\downarrow = + v_\downarrow$$

We can define the γ^5 eigenstates as LH or RH chiral states: u_R, u_L, v_R, v_L

$$\gamma^5 u_R = + u_R, \quad \gamma^5 u_L = - u_L, \quad \gamma^5 v_R = - v_R, \quad \gamma^5 v_L = v_L$$

If $E \gg m$: $u_R \to u_\uparrow$, $u_L \to u_\downarrow$, $v_R \to v_\uparrow$, $v_L \to v_\downarrow$

In general: solutions of the Dirac equation that are eigenstates of γ^5 are identical to the massless helicity states

Projection operators:

$$P_R = \frac{1}{2}(1+\gamma^5) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$P_L = \frac{1}{2}(1-\gamma^5) = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$P_R \cdot u_R = +u_R$, $P_L u_L = u_L$, $P_R u_L = 0$, $P_L u_R = 0$

$P_R v_R = 0$, $P_L v_L = 0$, $P_R v_L = v_L$, $P_L v_R = v_R$

P_R projects out only RH particle and LH antiparticle states

$\Rightarrow$ Any spinor can be written as: $\psi = \underbrace{\psi_R + \psi_L}_{\text{chiral eigenstates}} =$

$$= \frac{1}{2}(1+\gamma^5)\psi + \frac{1}{2}(1-\gamma^5)\psi$$

Chirality in QED

$ie\bar{\Psi}\gamma^5\phi \rightarrow$ interaction between a fermion and a photon represented by Dirac spinors Ψ and ϕ

We can decompose it into:

$$ie\bar{\Psi}\gamma^\mu\phi = ie(\bar{\Psi}_L + \bar{\Psi}_R)\gamma^\mu(\phi_L + \phi_R) =$$

$$= ie(\bar{\Psi}_R\gamma^\mu\phi_R + \bar{\Psi}_L\gamma^\mu\phi_L + \bar{\Psi}_R\gamma^\mu\phi_L + \bar{\Psi}_L\gamma^\mu\phi_L)$$

$$(\gamma^5)^2 = I, \ \gamma^{5+} = \gamma^5, \ \gamma^5\gamma^\mu = -\gamma^5\gamma^\mu$$

One can directly get: $\bar{\Psi}_L\gamma^\mu\phi_R = \bar{\Psi}_R\gamma^\mu\phi_L = 0$ (6.4.1 in the Thomson book)

$$P_R\gamma^\mu = \gamma^\mu P_L \quad , \quad P_R \cdot P_L = 0$$

$$\bar{u}_L\gamma^\mu u_R = \bar{u}_R\gamma^\mu u_L = \bar{v}_R\gamma^\mu v_L = \bar{v}_L\gamma^\mu v_R = \bar{v}_L\gamma^\mu u_L = \bar{v}_R\gamma^\mu u_R = 0$$

Helicity and chirality

helicity – projection of the spin of a particle onto its direction of motion

chirality – eigenstates of the γ^5 matrix

Relationship between helicity and chirality:
- decompose the helicity eigenstates onto their chiral components

Example

$$u_\uparrow(p,\theta,\varphi) = N\begin{pmatrix} c \\ s e^{i\varphi} \\ k\cdot c \\ k\cdot s\cdot e^{i\varphi} \end{pmatrix} \qquad k = \frac{p}{E+m}, \; N = \sqrt{E+m}$$

$$c = \cos\theta/2, \quad s = \sin\theta/2$$

$$P_R u_\uparrow = \frac{1}{2}(1+k)\cdot N \underbrace{\begin{pmatrix} c \\ s e^{i\varphi} \\ c \\ s e^{i\varphi} \end{pmatrix}}_{u_R}, \quad P_L u_\uparrow = \frac{1}{2}(1-k)\cdot N \underbrace{\begin{pmatrix} c \\ s e^{i\varphi} \\ -c \\ -s e^{i\varphi} \end{pmatrix}}_{u_L}$$

$$\Rightarrow u_\uparrow(p,\theta,\varphi) = \frac{1}{2}(1+k)\,u_R + \frac{1}{2}(1-k)\,u_L \quad (k\to 1,\; u_\uparrow \to u_R)$$

with $\gamma^5 u_R = +u_R$, $\gamma^5 u_L = -u_L$

Leading to: (for $E >> m$)

$$\bar{u}_{\downarrow}\gamma^{\mu}u_{\uparrow} = \bar{u}_{\uparrow}\gamma^{\mu}u_{\downarrow} = \bar{v}_{\downarrow}\gamma^{\mu}v_{\uparrow} = \bar{v}_{\uparrow}\gamma^{\mu}v_{\downarrow} = \bar{v}_{\downarrow}\gamma^{\mu}u_{\downarrow} = \bar{v}_{\uparrow}\gamma^{\mu}u_{\uparrow} = 0$$

helicity is conserved in high-energy interactions!!